

Energy confinement for a relativistic magnetic flux tube in the ergosphere of a Kerr black hole

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Abstract

In the MHD description of plasma phenomena the concept of magnetic field lines frozen into the plasma turns out to be very useful. We present here a method of introducing Lagrangian coordinates into relativistic MHD equations in general relativity, which enables a convenient mathematical formulation for the behaviour of flux tubes. With the introduction of these Lagrangian, so-called “frozen-in” coordinates, the relativistic MHD equations reduce to a set of nonlinear 1D string equations, and the plasma may therefore be regarded as a gas of nonlinear strings corresponding to flux tubes. Numerical simulation shows that if such a tube/string falls into a Kerr black hole, then the leading portion loses angular momentum and energy as the string brakes, and to compensate for this loss, momentum and energy is radiated to infinity to conserve energy and momentum for the tube. Inside the ergosphere the energy of the leading part turns out to be negative after some time, and the rest of the tube then gets energy from the hole. In our simulations most of the compensated positive energy is also localized inside the ergosphere because

the inward speed of the plasma is approximately equal to the velocity of the MHD wave which transports energy outside. Therefore, an additional physical process has to be included which can remove energy from the ergosphere. Magnetic reconnection seems fills this role releasing Maxwellian stresses and producing a relativistic jet.

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1. Introduction

Black holes are almost certainly the central engines of quasars, active galactic nuclei, and probably gamma ray bursts, and therefore the mechanism by which energy is extracted from them is of great astrophysical interest. While the exact form of this mechanism is not known, perhaps one of the best candidate was proposed by Blanford and Znajek [1], which in its simplest form involves an axisymmetric force-free electromagnetic configuration. The present paper considers an alternative approach to the Blanford-Znajek process. This is briefly described in the note [2], which is a variant of the Penrose mechanism [3] for extracting energy from a rotating black hole. Our argument relies on a very neat use of the analogy between the equations of relativistic magnetohydrodynamics (RMHD) and of a gas of strings. It is a fact that plasma embedded in a magnetic field can be considered as a collection of flux tubes, and each flux tube turns out to behave as a non-linear string. Therefore we can investigate the MHD flow as the time evolution of a test flux tube similar to the test particle approach [4]. Such an approach can provide clear physical meaning to the otherwise difficult problem of analyzing the dynamics of space plasmas (see, for example, [4–10]).

Following these examples we shall investigate the motion of a relativistic flux tube in the gravitational field of a Kerr black hole. The relativistic equations for a flux tube have been discussed in the papers [2,8]. Our analysis will show that a single flux tube can extract energy from a rotating black hole.

This paper is organized as follows: In Sections 2 and 3 we first introduce Lagrangian coordinates in the non-relativistic MHD equations, and then extend this to the case of general relativity. The conservation laws and gauge condition are considered in Section 4. A qualitative scheme for a flux tube interacting with a Kerr black hole is presented in Section 5. The numerical algorithm and results of the calculations are described in Sections 6 and 7. Test particle-string comparison is discussed in Section 8, and Section 9 is devoted to the summary and discussion.

2. Thin flux tube approximation

In order to study the time evolution of a flux tube, mathematically, we need first of all the underlying equations. To this end we will introduce Lagrangian coordinates into the MHD equations to obtain a convenient mathematical formulation for the flux tube motion [2].

To illustrate the general idea we will first study the simpler non-relativistic MHD case. The equation

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla \left(p + \frac{\mathbf{B}^2}{8\pi} \right) + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B}, \quad (0.1)$$

describes motion of a plasma embedded in a magnetic field. Here $\rho, p, \mathbf{v}, \mathbf{B}$ are plasma density, pressure, velocity and magnetic field, respectively. To distinguish a flux tube it is an advantage to rewrite equation (0.1) in terms of Lagrangian coordinates, using the frozen-in condition [11]:

$$\frac{\partial}{\partial t} \frac{\mathbf{B}}{\rho} + (\mathbf{v} \cdot \nabla) \frac{\mathbf{B}}{\rho} = \left(\frac{\mathbf{B}}{\rho} \cdot \nabla \right) \mathbf{v}, \quad (0.2)$$

Let us consider a small fragment δr of a flux tube with cross section δS . The mass of the plasma δM as well as the magnetic flux δF_B in this fragment have to be conserved according to equation (0.2) [11]:

$$\delta M = \rho \delta r \delta S = \text{const}, \quad \delta F_B = B \delta S = \text{const}. \quad (0.3)$$

Hence, the mass of plasma in the fragment of the tube with unit flux $\delta \alpha$ is also conserved:

$$\delta \alpha = \frac{\rho \delta r}{B} = \text{const}, \quad (0.4)$$

and, consequently, we can measure the length of the tube in units of mass α instead of the usual Cartesian coordinates.

As a second variable we can choose the Lagrangian time τ . It can be verified that the following relations are valid in the new, so-called frozen-in coordinates τ, α [4]:

$$\frac{\partial \mathbf{r}}{\partial \alpha} = \frac{\mathbf{B}}{\rho}, \quad \frac{\partial}{\partial \alpha} = \frac{\partial}{\partial \mathbf{r}} \cdot \frac{\partial \mathbf{r}}{\partial \alpha} = \left(\frac{\mathbf{B}}{\rho} \cdot \nabla \right), \quad (0.5)$$

$$\frac{\partial \mathbf{r}}{\partial \tau} = \mathbf{v}, \quad \frac{\partial}{\partial \tau} = \frac{\partial t}{\partial \tau} + \frac{\partial}{\partial \mathbf{r}} \cdot \frac{\partial \mathbf{r}}{\partial \tau} = \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla), \quad (0.6)$$

consequently,

$$\left(\frac{\mathbf{B}}{\rho} \cdot \nabla \right) \rightarrow \frac{\partial}{\partial \alpha}, \quad (0.7)$$

$$\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \rightarrow \frac{\partial}{\partial \tau}. \quad (0.8)$$

Bearing in mind relations (0.7,0.8) we can rewrite the equation of motion (0.1) in terms of the frozen-in coordinates τ, α :

$$\frac{\partial^2 \mathbf{r}}{\partial \tau^2} - \frac{1}{4\pi} \frac{\partial}{\partial \alpha} \left(\frac{\partial(\rho \mathbf{r})}{\partial \alpha} \right) = -\frac{1}{\rho} \nabla P(\mathbf{r}). \quad (0.9)$$

Generally speaking, the total pressure $P(\mathbf{r})$ is unknown in advance, but fortunately there are a set of problems when $P(\mathbf{r})$ can be considered as a given function. Then, the plasma density ρ is determined from the following nonlinear equation:

$$P(\mathbf{r}) = p + \frac{B^2}{8\pi} = c_e \rho^\kappa + \frac{1}{8\pi} \rho^2 \left(\frac{\partial \mathbf{r}}{\partial \alpha} \right)^2, \quad (0.10)$$

which is just definition of the total (gas plus magnetic) pressure. Here κ is the polytropic index, and c_e is the entropy of a fluid element which is conserved during plasma motion:

$$\frac{\partial}{\partial \tau} \left(\frac{p}{\rho^\kappa} \right) = 0. \quad (0.11)$$

The MHD equation of motion (0.1), written in frozen-in coordinates (0.9) is the equation of a non-linear string, which is well known in mathematical physics and plays an important role in the description of many wave-like processes. Therefore we can find an analogy between the non-linear string and the magnetic flux tube. Moreover, we can think of a plasma embedded in a magnetic field as a collection of non-linear strings (flux tubes) rather than a collection of fluid elements.

There is another way to introduce an analogy between the magnetic flux tube and the non-linear string, using a variation method. We can write the action functional S for a flux tube embedded in a pressure field $P(\mathbf{r})$ as the difference between kinetic and potential energy:

$$S = \int \left[\frac{1}{2} \left(\frac{\partial \mathbf{r}}{\partial \tau} \right)^2 - \frac{1}{4\pi} \left(\rho \frac{\partial \mathbf{r}}{\partial \alpha} \right)^2 - \varepsilon - \frac{P(\mathbf{r})}{\rho} \right] d\alpha d\tau, \quad (0.12)$$

where ε is the internal energy of the plasma. The action (0.12) for a flux tube is similar to that for a non-linear string. It can be used to derive conservation laws corresponding to a cyclic variable such as energy or momentum conservation for a tube.

To establish a bridge between non-relativistic string equations and those in general relativity it is advantageous to write equations (0.9) in terms of arbitrary coordinates ($x^i = x^1, x^2, x^3$) with line element $ds^2 = g_{ik} dx^i dx^k$ using the variational method:

$$\begin{aligned} \frac{\partial^2 x^l}{\partial \tau^2} + \Gamma_{ik}^l \frac{\partial x^i}{\partial \tau} \frac{\partial x^k}{\partial \tau} - \frac{\partial}{\partial \alpha} \left(\frac{\rho}{4\pi} \frac{\partial x^l}{\partial \alpha} \right) \\ - \frac{\rho}{4\pi} \Gamma_{ik}^l \frac{\partial x^i}{\partial \alpha} \frac{\partial x^k}{\partial \alpha} = - \frac{g^{il}}{\rho} \frac{\partial P}{\partial x^i}, \end{aligned} \quad (0.13)$$

where Γ_{ik}^l is the Christoffel symbol.

Let us now examine under what circumstances a test flux tube may represent MHD flow as a whole. It is well known that characteristics of the string equations (0.9-0.11) are Alfvénic and slow waves [12], respectively:

$$\frac{d\alpha}{d\tau} = \pm \sqrt{\frac{\rho}{4\pi}}, \quad \frac{d\alpha}{d\tau} = \pm \sqrt{\frac{\kappa \rho p}{4\pi(2P + p(\kappa - 2))}}. \quad (0.14)$$

Hence, the fast wave which is also characteristic of the general MHD system of equations [11], is left out in the string approach. The physical reason for this is clear: the fast wave is produced by gradients in the total pressure, but $P(\mathbf{r})$ is fixed in equations (0.9-0.11). There is therefore no driving mechanism for fast waves.

Therefore, MHD problems which involve significant variations of total pressure such as gasdynamic-type explosion, are unlikely to be applicable using the string equations (0.9-0.11). But processes of accumulation and relaxation of Maxwellian tensions can very often be

described by the string approach since the total pressure does not seem to vary appreciably. Hence, $P(\mathbf{r})$ may be considered as a given function or at least may be determined by a perturbation method [4,7,12].

The full set of MHD equations is difficult to solve in general, especially for 3D time-dependent problems, but application of the string method may lead to a considerable simplification in certain problems. In particular, for those problems in which accumulation and relaxation of Maxwellian stresses play the dominant role under nearly constant total pressure conditions, the general 4D MHD problems can be reduced to investigation of the behaviour of a 1D test flux tube/string. Such a method has been successfully applied to problems in magnetospheric [5,7,12], solar [6,9] and astrophysical [2,10] plasmas.

Here we apply this idea to the accretion of magnetized plasma on a rotating black hole.

3. Frozen-in coordinates in general relativity

To derive the string equations appropriate to the theory of general relativity, we have to introduce lagrangian co-ordinates into relativistic magnetohydrodynamic (RMHD) equations which can be presented as follows [13]:

$$\nabla_i \rho u^i = 0, \quad (0.15)$$

$$\nabla_i T^{ik} = 0, \quad (0.16)$$

$$\nabla_i (h^i u^k - h^k u^i) = 0. \quad (0.17)$$

Here equation (0.15) is the continuity equation, equation (0.16) is the energy-momentum equation, and equation (0.17) is Maxwell's equation; u^i is the time-like vector of the 4-velocity, $u^i u_i = 1$,

$$h^i = *F^{ik} u_k \quad (0.18)$$

is the space-like 4-vector of the magnetic field, $h^i h_i < 0$, $*F^{ik}$ is the dual tensor of the electromagnetic field, and T^{ij} is the stress-energy tensor:

$$T^{ij} = Qu^i u^j - Pg^{ij} - \frac{1}{4\pi} h^i h^j, \quad (0.19)$$

where

$$P \equiv p - \frac{1}{8\pi} h^k h_k, \quad Q \equiv p + \varepsilon - \frac{1}{4\pi} h^k h_k. \quad (0.20)$$

Here p is the plasma pressure, P is the total (plasma plus magnetic) pressure, ε is the internal energy including ρc^2 , and g_{ik} is the metric tensor with signature $(1, -1, -1, -1)$.

It is worth noticing that the 4-vector of the magnetic field is orthogonal to the 4-velocity:

$$u_i h^i = 0, \quad (0.21)$$

which is evident because $*F^{ik}$ is an antisymmetric tensor.

The previous analysis and discussion raises the question whether we can introduce a coordinate system in such a way that the trajectory u^i and magnetic field lines h^i become coordinate lines, like α and τ in non-relativistic MHD. The answer is provided by Lie analysis, which states that a necessary and sufficient condition for two vectors a^i and b^i to act as coordinate lines is that they satisfy the following Lie equation [14,15]:

$$a^i \nabla_i b^k = b^i \nabla_i a^k. \quad (0.22)$$

Hence, we have to rewrite the frozen-in equation (0.17) in the form of the Lie equation (0.22). Generally speaking, $\nabla_i h^i \neq 0$, but we can find a function q such that

$$\nabla_i q h^i = 0. \quad (0.23)$$

Then using (0.15) the Maxwell equation (0.17) can be rewritten in the form of a Lie derivative:

$$\frac{h^i}{\rho} \nabla_i \frac{u^k}{q} = \frac{u^i}{q} \nabla_i \frac{h^k}{\rho}, \quad (0.24)$$

and we can therefore introduce coordinates τ, α such that:

$$x_\tau^i \equiv \frac{\partial x^i}{\partial \tau} = \frac{u^i}{q}, \quad x_\alpha^i \equiv \frac{\partial x^i}{\partial \alpha} = \frac{h^i}{\rho} \quad (0.25)$$

with new coordinate vectors $u^i/q, h^i/\rho$ tracing the trajectory of a fluid element and the magnetic field in a flux tube. This implies that we define a magnetic flux tube as a flux tube of vector field h^i , and a bundle of trajectories as a flux tube of vector field u^i . Bearing in mind the normalization $u^i u_i = 1$ it is clear that the function q obeys:

$$q^2 g_{ik} x_\tau^i x_\tau^k = 1. \quad (0.26)$$

We can express the four-vectors of velocity and magnetic field in terms of 3-vectors \mathbf{v}, \mathbf{B} :

$$u^i = (\gamma, \gamma \frac{\mathbf{v}}{c}), \quad (0.27)$$

$$h^i = \left[\frac{\gamma(\mathbf{v} \cdot \mathbf{B})}{c}, \frac{\mathbf{B}}{\gamma} + \frac{\gamma(\mathbf{v} \cdot \mathbf{B})}{c^2} \right], \quad (0.28)$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor.

When $v \ll c$ equations (0.27,0.28) yield $u^i = (1, \frac{\mathbf{v}}{c}), h^i = (0, \mathbf{B})$, and the Lie equation (0.24) is reduced to the non-relativistic frozen-in equation (0.2). Therefore we can conclude that formally the Lagrangian relativistic coordinates τ, α (0.25) are the direct extension of non-relativistic frozen-in coordinates τ, α . It can be shown [8] that the mass coordinate α along the relativistic flux tube has the sense of a mass of the plasma for a tube with unit flux in the proper system of reference. The second coordinate τ is not any more Lagrangian or proper time, but is just a time-like parameter which traces the flux tube in the space-time of general relativity. The function q shows how much the parameter τ differs from the proper time t_p which is evident from comparison of the following equations:

$$\frac{\partial x^i}{\partial \tau} = \frac{u^i}{q}, \quad \frac{\partial x^i}{\partial t_p} = u^i. \quad (0.29)$$

The physical reason for this difference is that the proper time t_p depends on the fluid element. Hence, generally speaking, the condition $t_p = \text{const}$ can not define the flux tube as a whole, which consists of many fluid elements. The Lie equation (0.24) shows that there exists such a parameter τ which traces the flux tube in the space-time continuum of general relativity. We will refer to parameter τ as the string time.

So, we have shown that ideal version of the induction equation (0.17) together with the continuity equation (0.15) allow us to introduce the Lagrangian coordinates τ, α into the RMHD equations. The functions $x^i(\tau, \alpha)$ sweep the 2D worldsheet in the space-time continuum which consists of trajectories of the fluid elements for $\alpha = \text{const}$, and the magnetic field lines for $\tau = \text{const}$.

4. Variational method and string RMHD equations

Using (0.25) it is possible to derive the following analog of the non-relativistic relations (0.7, 0.8):

$$\frac{u^i}{q} \nabla_i = \frac{\partial}{\partial \tau}, \quad (0.30)$$

$$\frac{h^i}{\rho} \nabla_i = \frac{\partial}{\partial \alpha}. \quad (0.31)$$

Then, the energy-momentum equation (0.16) can be rearranged to form a set of string equations in terms of the frozen-in coordinates:

$$\begin{aligned} -\frac{\partial}{\partial \tau} \left(\frac{Qq}{\rho} x_\tau^l \right) - \frac{Qq}{\rho} \Gamma_{ik}^l x_\tau^i x_\tau^k \\ + \frac{\partial}{\partial \alpha} \left(\frac{\rho}{4\pi q} x_\alpha^l \right) + \frac{\rho}{4\pi q} \Gamma_{ik}^l x_\alpha^i x_\alpha^k = -\frac{g^{il}}{\rho q} \frac{\partial P}{\partial x^i}, \end{aligned} \quad (0.32)$$

where Γ_{ik}^l is the Christoffel symbol as before.

The string equations (0.32) for a flux tube embedded in a gravitational field $g_{ik}(x^i)$ and a pressure field $P(x^i)$ can also be derived from the action [8]:

$$S = - \int L(x_\tau^i, x_\alpha^i, x^i) d\tau d\alpha = - \int \frac{Q}{\rho} \sqrt{g_{ik} x_\tau^i x_\tau^k} d\tau d\alpha, \quad (0.33)$$

where L is the Lagrangian density.

The action (0.33) is invariant under τ -reparametrization $\tau \rightarrow \tau'(\tau)$. Therefore the canonical Hamiltonian

$$\frac{\partial L}{\partial x_\tau^i} x_\tau^i - L, \quad (0.34)$$

is easily seen to vanish identically which is always the case for systems with "time"-reparametrization invariance [16]. This implies that equations (0.32) are not independent, and we need a gauge condition to fix parametrization.

For simplicity we derive the gauge constraint for the particular case when entropy is initially uniform along the tube. Since entropy is conserved in the course of the motion [13], we can conclude that entropy does not depend on α for all τ . Hence, the Lagrangian (0.33) does not depend explicitly on α , which leads to the following equation:

$$\frac{\partial}{\partial \alpha} \left(\frac{\partial L}{\partial x_\alpha^i} x_\alpha^i - L \right) = - \frac{\partial}{\partial \tau} \left(\frac{\partial L}{\partial x_\tau^i} x_\alpha^i \right), \quad (0.35)$$

and we find that:

$$\frac{\partial}{\partial \alpha} \left(w \sqrt{g_{ik} x_\tau^i x_\tau^k} \right) = \frac{\partial}{\partial \tau} \left(\frac{Q}{\sqrt{g_{ik} x_\tau^i x_\tau^k}} g_{ik} x_\tau^i x_\alpha^i \right). \quad (0.36)$$

Here $w = \varepsilon + p/\rho$ is the enthalpy of the plasma.

Four vectors of velocity and magnetic field are orthogonal (0.21), hence the right hand side of the equation (0.36) has to vanish which immediately yields:

$$w \sqrt{g_{ik} x_\tau^i x_\tau^k} = f(\tau), \quad (0.37)$$

where $f(\tau)$ is an arbitrary function. It is appropriate to choose parametrization $\tau \rightarrow \tau'(\tau)$ so that $f(\tau) = 1$, which corresponds to the following gauge condition:

$$q = \sqrt{g_{ik} x_\tau^i x_\tau^k} = \frac{1}{w}. \quad (0.38)$$

Somewhat surprisingly it turns out that the function q which is responsible for the string time is defined by the plasma enthalpy.

Using the gauge condition (0.38) it can be shown that equations (0.32) are of hyperbolic type with relativistic alfvénic characteristics

$$\frac{d\alpha}{d\tau} = \pm \frac{\rho}{w \sqrt{4\pi Q}}, \quad (0.39)$$

and slow mode characteristics

$$\frac{d\alpha}{d\tau} = \pm \sqrt{\frac{\kappa \rho p}{4\pi w^3(2P + p(\kappa - 2))}} \quad (0.40)$$

similar to the non-relativistic case (0.14).

Ones again, by formulating the RMHD equations in terms of frozen-in coordinates, the energy-momentum equation (0.16) reduces to a set of 1D wave equations (0.32), which are in fact nonlinear string equations. The behaviour of flux tubes can therefore be studied through solving a set of string equations, and an analogy can be drawn between the behaviour of strings and flux tubes [2,4,7–10].

For a cyclic variable x^m there is a conservation law which states that:

$$\frac{\partial}{\partial \tau} \left(\frac{\partial}{\partial x_\tau^m} \left(\frac{Q}{q\rho} \right) \right) = - \frac{\partial}{\partial \alpha} \left(\frac{\partial}{\partial x_\alpha^m} \left(\frac{Q}{q\rho} \right) \right). \quad (0.41)$$

In the non-relativistic limit $v^2 \ll c^2$ the string equations (0.32), the action (0.33), and the equations of characteristics (0.39,0.40) are reduced to their analogous equations (0.9), (0.12), and (0.14), respectively.

5. Motion of a flux tube near a Kerr black hole

The Kerr metric in Boyer-Lindquist coordinates is given by the following line element [14]:

$$\begin{aligned} ds^2 = & \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 \\ & - (r^2 + a^2 + \frac{2Mra^2}{\Sigma} \sin^2 \theta) \sin^2 \theta d\varphi^2 \\ & + \frac{4Mra}{\Sigma} \sin^2 \theta d\varphi dt, \end{aligned} \quad (0.42)$$

where

$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2 \cos^2 \theta. \quad (0.43)$$

Here M and a are the mass and angular momentum of the hole, respectively, and we have used a system of units in which $c = 1$, $G = 1$.

Let us now consider a test flux tube which falls from infinity into a Kerr black hole. For cyclic variables t and φ , the energy and angular momentum conservation laws for the flux tube can be written as:

$$\int_{\alpha_1}^{\alpha_2} \frac{Q}{w\rho} (g_{tt}t_\tau + g_{t\varphi}\varphi_\tau) d\alpha = E, \quad (0.44)$$

$$\int_{\alpha_1}^{\alpha_2} \frac{Q}{w\rho} (g_{t\varphi}t_\tau + g_{\varphi\varphi}\varphi_\tau) d\alpha = -L, \quad (0.45)$$

if there is no flux of energy and angular momentum through the ends α_1, α_2 of the flux tube.

It is worth comparing the tube conservation laws (0.44, 0.45) with those for a test particle [14]:

$$g_{tt}t_\tau + g_{t\varphi}\varphi_\tau = E_p, \quad (0.46)$$

$$g_{t\varphi}t_\tau + g_{\varphi\varphi}\varphi_\tau = -L_p. \quad (0.47)$$

E_p and L_p differ from energy and angular momentum densities of the string in equations (0.44, 0.45) by just factor $Q/w\rho$, i.e., a fluid element can be considered as a particle.

Let us suppose to start off with that each element of the tube has no angular momentum at infinity. Hence, if there is no magnetic field, no tube element would interact with any other element, and as it falls into the black hole it would rotate with an angular velocity $\omega_0 = -g_{t\varphi}/g_{\varphi\varphi}$ of fiducial observers which follows from equation (0.47) [14].

Because of inhomogeneous rotation of the space around the black hole, the flux tube becomes stretched and twisted, and gravitational energy is partially converted into magnetic energy of the swirling tube. The strong magnetic field (string) evidently slows down rotation of the flux tube as it falls, hence this part of the tube with $\Omega_{tube} = \varphi_\tau/t_\tau < \omega_0$ must have negative momentum with respect to the rotation of the hole. On the other hand, angular momentum of the tube as a whole needs to be conserved (0.45), and to compensate positive angular momentum has to be radiated to infinity.

This is a continuous process: the deeper the tube falls in the hole, the more the tube is stretched, the stronger the magnetic field gets, the slower the tube rotates, the more negative momentum is generated, and the more positive momentum escapes to infinity.

Now we have to take into account the fact that the stretching and twisting of the falling flux tube is most pronounced close to, and especially inside, the ergosphere where $g_{tt} < 0$ and the energy of a particle or tube element can be negative [3,14].

As was pointed out previously, as the flux tube falls into the hole, the leading part progressively loses more and more angular momentum and energy due to string braking, and by the time it encounters the ergosphere boundary it must have a substantial amount of negative momentum and, hence, this leading part can have negative energy inside the ergosphere.

Since the energy of the tube as a whole has to be conserved (0.44), the part of the tube with positive energy ends up with an energy greater than the initial tube energy, E . To some extent this is similar to the Penrose process [3], but now we do not need to invoke the interaction or decay of particles or tubes, since just a single tube can extract energy from the hole.

6. Numerical method

The treatment of the previous section was based just on the energy and momentum conservation laws which can give only a rough description of the flux tube motion. The relativistic string is a highly nonlinear object, hence analytical methods can hardly provide necessary details. It seems that the only way out is the direct numerical simulation of a test flux tube motion in the vicinity of a rotating black hole.

To implement the numerical approach some preliminary work is needed.

6.1 Normalization. We normalize plasma density to its initial value ρ_0 , length to the radius of the ergosphere in the equatorial plane r_g , velocity to the light speed c , time scale to the ratio r_g/c , magnetic field to $\sqrt{4\pi\rho_0}c$, the plasma pressure and the energy density to $\rho_0 c^2$, α to $r_g\sqrt{\rho_0/(4\pi c^2)}$. Note that radius of the event horizon is equal to $1/2$ for an extreme rotating black hole.

6.2 Total pressure. The distribution of the total pressure in the vicinity of a black

hole seems to be mainly determined by the gravitational field. Since g_{ik} is axisymmetric we may expect that the function $P(x^i) = P(r, \theta)$ is also axisymmetric. Thus, we may expect significant increase of the total pressure to the event horizon where stretching of flux tubes is the most pronounced. Taking all this into account we choose for simulation the following expression:

$$P(r) = \frac{c_P}{(r - r_g)^d}, \quad (0.48)$$

where c_P and d are some constants. We run our programme for different distributions $P(x^i)$, and the results depend weakly on this function. The black hole is such a powerful object that qualitatively the behaviour of a string mainly depends on the characteristics of the black hole itself (angular momentum, first of all) rather than details of pressure distribution. However, there are two important restrictions on $P(x^i)$. It should not be too big to exceed the gravitational attraction, and it can not be too small because magnetic field strength is limited by the total pressure (0.20).

6.3 String equations. From the numerical point of view it is convenient to rewrite the string equations (0.32) in conservative form using the variational method:

$$\begin{aligned} \frac{\partial}{\partial \tau} \left(\frac{Qw}{\rho} x_{i\tau} \right) - \frac{\partial}{\partial \alpha} \left(\frac{\rho}{4\pi w} x_{i\alpha} \right) &= \frac{1}{\rho w} \frac{\partial P}{\partial x^i} \\ &+ \frac{1}{2} \frac{\partial g_{jk}}{\partial x^i} \left(\frac{Qw}{\rho} x_\tau^j x_\tau^k - \frac{\rho}{w} x_\alpha^j x_\alpha^k \right), \end{aligned} \quad (0.49)$$

where $x_{i\tau} \equiv g_{ik} x_\tau^k$, $x_{i\alpha} \equiv g_{ik} x_\alpha^k$. The equations (0.49) have an advantage in comparison with (0.32) in that energy density (0.44) and angular momentum density (0.45) are explicitly included in the numerical scheme.

Adding the following equations:

$$\frac{\partial x_\alpha^i}{\partial \tau} = \frac{\partial x_\tau^i}{\partial \alpha}, \quad (0.50)$$

$$\frac{\partial x^i}{\partial \tau} = x_\tau^i, \quad (0.51)$$

we can present the string equations in terms of 12-dimensional vectors of state W , flux F and source S :

$$\frac{\partial W}{\partial \tau} - \frac{\partial F}{\partial \alpha} = S, \quad (0.52)$$

where

$$W = \left(\frac{Qw}{\rho} x_{i\tau}, x_{\alpha}^i, x^i \right), \quad (0.53)$$

$$F = \left(\frac{\rho}{w} x_{i\alpha}, x_{\tau}^i, 0 \right), \quad (0.54)$$

$$S = \left(\frac{1}{\rho w} \frac{\partial P}{\partial x^i} + \frac{1}{2} \frac{\partial g_{jk}}{\partial x^i} \left(\frac{Qw}{\rho} x_{\tau}^j x_{\tau}^k - \frac{\rho}{w} x_{\alpha}^j x_{\alpha}^k \right), 0, x_{\tau}^i \right). \quad (0.55)$$

We solved the string equation (0.52) numerically using the ordinary Lax–Wendroff method as well as TVD scheme.

6.4 Calculation of density. The plasma is assumed to be polytropic with the following equations of state for gas pressure, internal energy and enthalpy in dimensionless form:

$$p = c_e \rho^{\kappa}, \quad \varepsilon = 1 + \frac{c_e}{\kappa - 1} \rho^{\kappa-1}, \quad w = 1 + \frac{c_e \kappa}{\kappa - 1} \rho^{\kappa-1}. \quad (0.56)$$

Note that the relativistic term ρc^2 is taken into account, and, for example, the enthalpy per unit mass is equal to $w = c^2 + \frac{p}{\rho}$.

The plasma density is obtained from the following transcendental equation:

$$P(x^i) = c_e \rho^{\kappa} - \frac{1}{2} \rho^2 g_{ik} x_{\alpha}^i x_{\alpha}^k, \quad (0.57)$$

which is just a definition of the total pressure. For the particular case $\kappa = 2$ the density can be found analytically:

$$\rho = \sqrt{\frac{P(x^i)}{c_e - \frac{1}{2} g_{ik} x_{\alpha}^i x_{\alpha}^k}}. \quad (0.58)$$

For the sake of simplicity we will suppose the polytropic index to be 2.

6.5 Initial state. For the initial moment $\tau = 0$ the flux tube is assumed to be elongated along the x -axis outside the ergosphere:

$$x^1 = c_x \alpha, \quad x^2 = y_0, \quad x^3 = z_0, \quad (0.59)$$

where c_x, y_0, z_0 are constants. Then the plasma density is determined from equation (0.58).

We assume that the initial angular momentum of each element of the string vanishes, hence

$$\phi_\tau = -\frac{g_{t\phi}}{g_{\phi\phi}}t_\tau. \quad (0.60)$$

Then the initial velocity t_τ can be obtained from the gauge condition (0.38)

$$t_\tau = \frac{1}{w} \sqrt{\frac{g_{\phi\phi}}{g_{tt}g_{\phi\phi} - g_{t\phi}^2}}. \quad (0.61)$$

Using equations (0.59- 0.61) we can find W, F and S for the initial moment $\tau = 0$.

6.6 Boundary conditions. The length of the flux tube must be sufficiently long to avoid influence of energy and momentum flux through the edges of the string. We used free boundary conditions at the edges:

$$\frac{\partial}{\partial \alpha} x_\tau^i = 0, \quad (0.62)$$

and controlled conservation of the total energy and angular momentum of the string.

6.7 Tortoise variable. To avoid too big a step along the radial coordinate r we used the tortoise distance [14]

$$s = -\ln(r - r_h), \quad (0.63)$$

where r_h is the radius of the event horizon.

6.8 Error control. The problem under consideration is a stiff one, and at some stage errors of calculation started to grow exponentially. Therefore it is important to control accuracy of the numerical scheme. To this end we used the gauge condition (0.38) which has not been actually involved in the numerical method. If the difference

$$w\sqrt{g_{ik}x_\tau^i x_\tau^k} - 1 < \varepsilon_0 \quad (0.64)$$

exceeds some level ε_0 we stop the calculations. The value $\varepsilon_0 = 0.03$ has been taken in most of the runs.

6.9 Initial parameters. We present hereafter results of our simulation for the following parameters:

$$c_e = 0.5, \quad c_P = 0.01, \quad c_x = 1, \quad d = 2, \quad y_0 = 1.9, \quad z_0 = 1.4. \quad (0.65)$$

The number of grid points along the string $-20 < \alpha < 20$ is chosen to be 4000. It will be shown in Figures only the central part $-10 < \alpha < 10$. The black hole is supposed to be near the extreme rotation $a = .99M$.

7. Results of numerical simulation

7.1 Geometry of the falling flux tube. From the numerical simulation we obtain the functions $x^i(\tau, \alpha), x_{i\tau}(\tau, \alpha), x_\alpha^i(\tau, \alpha)$. The evolution of Boyer-Lindquist coordinates t, r, θ, ϕ which specify the position of the string as a function of α for different string times $\tau = \delta, 2\delta, \dots, 7\delta$ is shown in Figure 1, where $\delta = 5$. One can see that the central fragment of the string is attracted by the hole more strongly compared with the trailing parts. Hence, this fragment is stretched in the direction of the hole, and simultaneously the string becomes involved into rotation around the Kerr black hole. When the leading part encounters the outer boundary of the ergosphere, the static limit surface, both effects (attraction and rotation) become more pronounced. Eventually, the flux tube sweeps around the horizon producing a characteristic spiral structure.

To investigate the string geometry we embedded the string into 3D flat space using pseudo-Cartesian coordinates [17]:

$$\begin{aligned} x &= \sqrt{r^2 + r_g^2} \sin\theta \cos\phi, \\ y &= \sqrt{r^2 + r_g^2} \sin\theta \sin\phi, \\ z &= r \cos\theta. \end{aligned} \quad (0.66)$$

The mapping (0.66) corresponds to the case $M = 0$ (mass of the hole vanishes), and the Boyer-Lindquist coordinates describe a flat space.

One can observe in Figure 2 how the black hole attracts the string and makes it spin fast around the event horizon.

It is interesting that the motion of the string inside the ergosphere is rather complicated: the falling tube simultaneously slowly approaches the event horizon, quickly rotates, moves from the equator to the cusp region, is reflected, returns back to the equator and so on (Figure 1,2). Therefore the spiral structure of the string inside the ergosphere is very complicated.

In the course of time plasma flows into the ergosphere, but no fluid element can escape the latter.

7.2 Energy and angular momentum. The distribution of the energy density (first panel) and angular momentum density (second panel) along the falling tube for the same string times is shown in the second column of Figure 1.

Our simulations confirm the general conclusions of the previous section based on conservation laws and differential rotation. The leading fragment of the falling tube is located closer to the black hole, is more stretched, hence string braking acts more strongly. As a result this part of the tube has to rotate more slowly than space (i.e., fiducial observers) which implies that negative angular momentum has to be generated. However, the angular momentum of the whole tube which vanishes at the initial moment has to be conserved, and to compensate for this loss positive angular momentum appears and propagates along the tube (see Figure 1). In the course of time when the tube enters the ergosphere this effect becomes more and more pronounced, but the negative angular momentum is created most efficiently when the tube is deep down near the horizon.

It is apparent that such a behaviour of angular momentum should have some consequences also in the behaviour of the tube energy, but surprisingly it turns out that the angular momentum distribution is nearly proportional to the energy distribution along the string which can be seen by comparing the two practically identical pictures in Figures 1. This circumstance allows to include energy as a last term in the chain of physical processes related to the leading fragment of the tube:

Stretching \rightarrow Braking \rightarrow

Negative angular momentum \rightarrow Negative energy. (0.67)

The physical reason for this effect is that negative energy is created inside a narrow layer near the horizon where the time-coordinate is approximately proportional to the rotation (compare panels for $t(\tau, \alpha)$ and $\phi(\tau, \alpha)$ in Figure 1).

With regards to energy problem it was not clear in advance whether or not the flux tube gets negative energy inside the ergosphere, but our simulations show that in fact it does. As one can see in Figure 1, the energy density progressively decreases from the initial value ~ 1 to less than -20 at $\tau = 30$. Simultaneously plasma density increases from $\sim .05$ to about 180, and magnetic energy density $B = h^i h_i / 2$ increases from $\sim .05$ to ~ 100 (third and fourth panels in the second column in Figure 1).

The negative energy starts to be created approximately at $\tau = 10$ (see Figure 3) and then decreases linearly with string time. As a result, the part of the tube with positive energy gets extra energy from the black hole, and ends up with double the energy $E \sim 80$ at $\tau = 30$ in comparison with the total initial energy of the string $E_0 \sim 40$.

7.3 Energy confinement. The negative energy as well as the negative angular momentum are created in the narrow layer very near to the event horizon in the range $r_g = .51 < r < .52$, and moreover most of the compensated positive energy and angular momentum is also located in this region (see Figure 4a).

There is also a wave of positive E and L (see Figure 4b) which propagates towards the outer boundary of the ergosphere located at $r \sim 1$. Unfortunately, the speed of the wave is approximately equal to the velocity of the plasma entering the static limit surface, and as a result the wave can not leave the ergosphere. Both negative and most of the positive energy turns out to be localized near the black hole, and there is some kind of energy confinement inside the ergosphere.

Nevertheless, this confinement is not absolute. The wave with $E > 0$ can still carry some small amount of energy out of the ergosphere. For example, the energy density near the

static limit surface $r = 1$ at $\tau = 30$ is double the initial value (see Figure 4b), and very slowly increases with string time. Therefore the tube can extract energy from the ergosphere but at a relatively low rate compared to the negative energy creation rate. The paradox of the situation is that the falling string can effectively generate the extra positive energy by taking it from the rotating black hole, but on the other hand can not effectively transport the extra energy through the outer boundary of the ergosphere.

We run simulations with different initial parameters of the string such as position, total pressure behaviour, etc., but the main result stays the same: the extra energy of the falling tube is created near the event horizon at a much higher rate than that for the extraction of energy out of the ergosphere.

8. Test particle - flux tube comparison

It is interesting to compare the behaviour of a test particle with that of a tube/string. We noted previously that the formal expressions for the energy and angular momentum (0.44- 0.47) are similar up to a factor $Q/w\rho$. We can therefore identify a link between a test particle and a string. On the other hand there is also a major difference between them. In particular, test particles can be regarded as independent entities, while fluid elements closely are coupled to each other. Thus, E and L are conserved separately for each particle, but for the string the conservation laws hold for the tube as a whole, and we can not apply them to the individual fluid elements. This statement has several important implications:

- i. The Penrose mechanism [3], which is based on the test particle approach, requires at least two particles for exchange and transmission of energy and angular momentum. In the string approach, in contrast, a single flux tube is sufficient to extract energy from the black hole.
- ii. Generally, the trajectories of test particles and flux tubes differ significantly. Although the trajectories may start off the same initially if the magnetic field involved is small, eventually the effects of the magnetic field and associated stresses lead to completely different

behaviour. In our simulations, for example, we did not find any signatures of stable orbits.

iii. It is known that a test particle takes an infinite time to reach the horizon relative to an observer at infinity, even though from the point of view of the particle itself this take only a finite time and can even be rapid.

For the string, on the other hand, longer time intervals imply increasingly stronger magnetic stresses, as the string gets tightly wound around the event horizon. Thus, an infinite time interval would probably lead to an infinite amount of tension, and we would not expect stress to build up to this extent. To get round this problem, we need to invoke an additional process. Since the leading part of the string develops a loop, which gives rise to an antiparallel magnetic field configuration (see Figure 2), it would seem natural to suggest that reconnection could fulfill the role of this additional process. Reconnection is able to cut the fragment of the tube with negative energy, which corresponds to this loop, and thus release stresses which are building up, similar to what happens in current sheet configurations in the magnetospheric and solar context [5,18,19]. In the case of the solar-terrestrial interaction, we know that flux tubes filled with solar plasma flow around the obstacle formed by the Earth's magnetic field. This is accomplished through reconnection of magnetic field lines at the boundary between the solar wind and the magnetosphere. Reconnection releases the stresses which would otherwise build up as the solar flux tubes hang and drape around the magnetosphere [18]. Similarly, a flux tube which encounters a black hole gets wound up around the event horizon, as demonstrated by our simulations. The only way, it seems, to release the resulting stresses and capture some of the energy inside the ergosphere to transmit it to the outside world, is through reconnection. This results in part of the flux tube with negative energy falling into the black hole, and the other part can then transfer some of the positive energy extracted from the black hole outside the ergosphere. The mechanism we have describe here whereby a magnetic flux tube is captured by a black hole and then extracts and releases some of the energy contained in the rotation of the black hole to the outside world may therefore be viable one to explain such phenomena as astrophysical jets [1].

9. Discussion and conclusion

The main aim of this paper is to present an example of how a falling flux tube can extract energy from a Kerr black hole. According to our numerical simulations, due to stretching and braking, the energy of the leading part of the tube becomes negative. Since the total energy of the string is conserved, the rest of the tube gets extra energy from the hole, i.e., extracts spinning energy from the latter. So, our simulations confirm in general the prediction of [2] concerning magnetic flux tube - Kerr black hole coupling, but some unexpected features cropped up.

For a sufficiently strong rotation of the hole and for a sufficiently strong initial magnetic field of the falling tube, the latter can effectively generate negative energy, however the fragments of the string with both negative and compensated positive energy are localized inside the ergosphere. The energy extracted from the black hole is confined inside the ergosphere and almost does not show up outside, and probably can not produce such a phenomena as relativistic jets from the AGN.

There are several possibilities for removing the spinning energy from the hole more effectively.

First of all, so far we have only studied the initial phase of the flux tube motion inside the ergosphere of a Kerr Black hole. We were able to run programmes only for relatively short string-time intervals $\tau < 30$ before errors of the numerical scheme exceeded the limit 3%. This corresponds, for example, for a black hole of a solar mass to a time duration of only 0.3 ms which is far well outside astronomical scales. The relativistic flux tube is a strongly non-linear system, and theoretically we can not avoid the possibility that energy will be extracted much more effectively for later times. On the other hand, our results show no signatures of such a behaviour, and it seems reasonable to conclude that the negative energy creation rate will always be much bigger than that of energy extraction from the ergosphere. Of course, this is only true if we do not invoke some additional processe.

An alternative is to consider the behaviour of the total pressure, which we have to define a priori in the string approach. Certainly, we were not able to try all possible variants of the total pressure distribution, and some of them may lead to a more powerful process of energy removal from a black hole. There also might be a scenario in which the whole problem is essentially time-dependent. We can imagine that accreting magnetized plasma which consists of many flux tubes, can produce a pressure which increases with time due to period of a long rotation around the horizon which eventually pushes the plasma out of the ergosphere in a bursty explosive-like regime. We can not answer all these questions right now, and they left for future studies.

Finally, the most promising process which can produce relativistic jets seems to be magnetic reconnection. Our conclusion is based on the investigation of the Earth's magnetosphere, where we have the benefit of direct spacecraft measurements. It turns out that the energy of the magnetospheric substorm comes from the magnetic energy stored in the magnetotail which is released in an explosive way by virtue of reconnection producing narrow flows of accelerating plasma [18]. The reconnection events at the magnetopause [18,20] (boundary of the magnetosphere) and solar flares [19] present other examples of this explosive energy release, when magnetic energy is first accumulated and then reconnection converts it into kinetic and internal energy to produce jets of accelerated plasma.

Therefore, we believe that our simulations reproduce the initial stage corresponding to the storage of the magnetic energy inside the ergosphere, which then creates conditions for a bursty regime of energy release involving reconnection. Inside the ergosphere parts of the flux tube with oppositely directed magnetic field are very close to each other (see Figure 2) and therefore reconnection [21,22] can come into play. The fragment of the falling flux tube with negative energy is absorbed by the hole, but the other one receives extra energy from the hole, thus producing a relativistic jet. Physically, plasma is accelerated to relativistic velocities by slow shocks which propagate along the tube and release Maxwellian tension. It is very important to note that acceleration does not happen in one instant, but it occurs continuously as the shocks propagate along the magnetic field. So, the initial

quasi-stationary regime generates Maxwellian tension along the string, and subsequently reconnection releases this tension to give rise to a relativistic jet.

Although reconnection can occur at any point near the black hole, only when it occurs inside the ergosphere can energy be extracted from the hole, and thus accelerate plasma to the relativistic velocities needed for cosmic jets. The stability of cosmic jets are explained in our model in terms of the relativistic velocities of the plasma and the spiral structure of the magnetic field.

It is worthwhile discussing the string model in relation to the Blandford-Znajek mechanism [1,23], which currently seems to be the most popular one. The string process is inherently three dimensional and time-dependent because it is based on differential rotation even for the quasi-stationary regime, although the magnetic field structure in the final stage may be nearly axisymmetric. It is well known that for a purely steady-state axisymmetric configuration, the flux surface must have rigid rotation [24] which is not true for the string mechanism. There is only a tendency to rigid rotation, which is clear from the redistribution of the angular momentum, i.e., the leading part of the falling tube spins more slowly, the trailing part faster. But to establish exactly rigid rotation takes an infinite time, which can not happen in reality. Therefore, the string mechanism differs from the Blandford-Znajek one based on steady-state and axisymmetrical patterns. Besides, the Blandford-Znajek process needs a magnetic field embedded in the hole's event horizon while the flux tube in the string approach does not reach the horizon at all so far, which emphasizes the difference even more.

Nevertheless, there are also common features for both mechanisms. First of all the main idea of the Blandford and Znajek mechanism [1,23] is that magnetic forces are the most appropriate to couple the black hole's spin to external matter. In other words, these mechanisms are the non-local variants of the original Penrose process [3] which make use of electromagnetic effects to remove spin energy from a Kerr black hole. Therefore we believe that future development could reveal that the Blandford-Znajek mechanism and the string model are just two different ways for describing the same physical process, even though they

now look rather different.

There is a direct generalization of the scheme described above to cosmic strings. It is a fact that formally the equations of motion for ordinary cosmic strings generated by the Nambu-Goto action [16] are the same as those for the flux tubes (0.32) if we replace the factors Qq/ρ and $\rho/4\pi q$ with an energy per unit length μ , estimated as 10^{20} kg/m . The only important difference is the gauge conditions, but they do not influence the energy and momentum conservation laws. There is an even closer analogy with the current-carrying strings, for which the energy per unit length and the string tension are in general different [25]. Therefore, we can expect extraction of spin energy of the hole by a cosmic string in the form of a spiral wave, and then reconnection can release the string tension to produce a fast moving part of the cosmic string.

Accretion with differential rotation leads to a general pattern with a spiral structure of the magnetic field and a plasma flow which eventually may result in jet formation by virtue of reconnection. The prediction of such a pattern is based just on the application of energy and angular momentum conservation for the flux tube, and there is nearly no dependence on the initial configuration of the plasma flow and magnetic field; the only restriction is that they should not be axisymmetric.

We believe that the string mechanism described above can be used to develop model of active galactic nuclei.

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FIGURES

FIG. 1. Numerical results for a flux tube falling onto a Kerr black hole as functions of the mass parameter α . Distributions of different parameters along the string are shown for the string times $\tau = \delta, 2\delta, \dots, 7\delta$ where $\delta = 5$. Left column: distributions of the Boyer-Lindquist coordinates t, r, θ, ϕ . Right column: distribution of the density of the energy, of the angular momentum, of the plasma, and of the magnetic energy.

FIG. 2. Visualization of the flux tube geometry by mapping Boyer-Lindquist coordinates to pseudo-Cartesian coordinates for the initial stage $\tau = \delta, 2\delta, \dots, 9\delta$, where $\delta = 1.2$ (top) and for the late string time $\tau = 30$ (bottom). The fragment of the string with negative energy is marked with red points.

FIG. 3. Behaviour of the total (solid), negative (dash-dotted), and positive (dotted) energy of the string in the course of string time.

FIG. 4. Distribution of the energy density of the string as a function of r (a) in the narrow layer near the horizon and (b) in the vicinity of the ergosphere. The fragment of the tube with $\alpha < 0$ is marked with points.









